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Energy Distribution in 2d Stringy Black Hole Backgrounds

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Abstract

We utilize Møller's and Einstein's energy-momentum complexes in order to explicitly evaluate the energy distributions associated with the two-dimensional "Schwarzschild" and "Reissner-Nordström" black hole backgrounds. While Møller's prescription provides meaningful physical results, Einstein's prescription fails to do so in the aforementioned gravitational backgrounds. These results hold for all two-dimensional static black hole geometries. The results obtained within this context are exploited in order Seifert's hypothesis to be investigated.

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Introduction

One of the most interesting problems which remains unsolved since the birth of General Theory of Relativity, is the energy-momentum localization. Many renowned physicists have been working on this problematic issue with Einstein to be first in the row. After Einstein's seminal work [1] on energy-momentum complexes a large number of expressions for the energy distribution were proposed [2,3,4,5,6,7]. These expressions were restricted to evaluate energy distribution in quasi-Cartesian coordinates. Møller [8] proposed a new expression for an energy-momentum complex which could be utilized to any coordinate system. However, the idea of the energy-momentum complex was severely criticized for a number of reasons. Firstly, although a symmetric and locally conserved object, its nature is nontensorial and thus its physical interpretation seemed obscure [9]. Secondly, different energy-momentum complexes could yield different energy distributions for the same gravitational background [10, 11]. Thirdly, energy-momentum complexes were local objects while there was commonly believed that the proper energy-momentum of the gravitational field was only total, i.e. it cannot be localized [12]. For a long time, attempts to deal with this problematic issue were made only by proposers of quasi-local approach [13, 14].

In 1990 the concept of energy-momentum complexes and their use for explicitly evaluating the energy distribution in given gravitational backgrounds was revived by Virbhadra [15, 16, 17]. At the same time Bondi [18] sustained that a nonlocalizable form of energy is not admissible in relativity so its location can in principle be found. Since then, numerous works on evaluating the energy distribution of several gravitational backgrounds have been completed employing the abandoned for a long time approach of energy-momentum complexes [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38]. In 1996 Aguirregabiria, Chamorro and Virbhadra [39] showed that five different[2] energy-momentum complexes yield the same energy distribution for any Kerr-Schild class metric. Additionally their results were identical with the results of Penrose [40] and Tod [41] using the notion of quasi-local mass. In 1999 Chang, Nester and Chen [42] proved that every energy-momentum complex is associated with a Hamiltonian boundary term. Thus, the energy-momentum complexes are quasi-local and acceptable.

In this paper we evaluate the energy distribution in two-dimensional static black hole backgrounds utilizing the Einstein's and Møller's prescriptions. Our interest in the two-dimensional gravitational backgrounds stems to the fact that lower dimensional theories

[2]Later on Virbhadra [44] came to know that Tolman's and Einstein's complexes which had been used in [39] were exactly the same (see footnote 1 in [44]).

of gravity provide simplified contexts in which to study various physical phenomena [43]. The choice of Einstein's approach among others was due to a recent work of Virbhadra in which he points out that the description of Einstein is the best one [44]. The reasons for presenting here the Møller's description are: (a) the argument that it is not restricted to quasi-Cartesian coordinates and (b) a work of Lessner [45] who argues that the Møller's energy-momentum complex is a powerful concept of energy and momentum in General Theory of Relativity.

The remainder of the paper is organized as follows. In Section 1 we present the concept of energy-momentum complexes in the framework of General Theory of Relativity. In Section 2 the expressions of the Einstein's energy-momentum complexes in four and two dimensional gravitational backgrounds are given. It is shown that for the case of two-dimensional static gravitational backgrounds the energy-momentum complex of Einstein cannot give meaningful physical results. In Section 3 we present the Møller's energy-momentum complexes in four and two dimensional gravitational backgrounds. An explicit expression for the energy-momentum complex of Møller in a two-dimensional static gravitational background is derived. In Section 4 we use the Møller's prescription in order to obtain the energy distribution in a two-dimensional "Schwarzschild" black hole background [46, 47]. The results extracted in three different coordinate systems (gauges) for the energy distribution in the above-mentioned gravitational background are identical. In Section 5 the energy distribution in a two-dimensional "Reissner-Nordström" black hole [48, 49] in Møller's prescription is explicitly calculated in "Schwarschild" gauge. The Seifert's hypothesis [50] is addressed in both Sections 4 and 5. Finally, Section 6 is devoted to a brief summary of results and concluding remarks.

1 Energy-Momentum Complexes

The conservation laws of energy and momentum for an isolated (closed), i.e. no external force acting on the system, physical system in the Special Theory of Relativity are expressed by a set of differential equations. Defining T^{μ}_{ν} as the symmetric energy-momentum tensor of matter and all non-gravitational fields the conservation laws are given by

$$T^{\mu}_{\nu,\,\mu} \equiv \frac{\partial T^{\mu}_{\nu}}{\partial x^{\mu}} = 0 \tag{1}$$

where

$$\rho = T^{t}_{t} \qquad j^{i} = T^{i}_{t} \qquad p_{i} = -T^{t}_{i} \tag{2}$$

are the energy density, the energy current density, the momentum density, respectively, and Greek indices run over the spacetime labels while Latin indices run over the spatial coordinate values.

Making the transition from the Special to General Theory of Relativity one adopts a simplicity principle which is called principle of minimal gravitational coupling. As a result of this, the conservation equation is now written as

$$T^{\mu}_{\nu;\,\mu} \equiv \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}\,T^{\mu}_{\nu}\right) - \Gamma^{\kappa}_{\nu\lambda}T^{\lambda}_{\kappa} = 0 \tag{3}$$

where g is the determinant of the metric tensor $g_{\mu\nu}(x)$. The conservation equation may also be written as

$$\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}\,T^{\mu}_{\nu}\right) = \xi_{\nu} \tag{4}$$

where

$$\xi_{\nu} = \Gamma^{\kappa}_{\nu\lambda}T^{\lambda}_{\kappa} \tag{5}$$

is a nontensorial object. For $\nu = t$ this means that the matter energy is not a conserved quantity for the physical system[3]. From a physical point of view this lack of energy conservation can be understood as the possibility of transforming matter energy into gravitational energy and vice versa. However, this remains a problem and it is widely believed that in order to be solved one has to take into account the gravitational energy [2, 4, 7, 8] .

By a well-known procedure, the nontensorial object ξ_{ν} can be written as

$$\xi_{\nu} = -\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}\,\vartheta^{\mu}_{\nu}\right) \tag{6}$$

where ϑ^{μ}_{ν} are certain functions of the metric tensor and its first order derivatives. Therefore, the energy-momentum tensor of matter T^{μ}_{ν} is replaced by the expression

$$\theta^{\mu}_{\nu} = \sqrt{-g}\,(T^{\mu}_{\nu} + \vartheta^{\mu}_{\nu}) \tag{7}$$

which is called energy-momentum complex since it is a combination of the tensor T^{μ}_{ν} and a pseudotensor ϑ^{μ}_{ν} which describes the energy and momentum of the gravitational field. The energy-momentum complex satisfies a conservation law in the ordinary sense, i.e.

$$\theta^{\mu}_{\nu,\,\mu} = 0 \tag{8}$$

[3] It is possible to restore the conservation law by introducing a local inertial system for which at a specific spacetime point $\xi_{\nu} = 0$ but this equality by no means holds in general.

and it can be written as

$$\theta_\nu^\mu = \chi_{\nu,\lambda}^{\mu\lambda} \tag{9}$$

where $\chi_\nu^{\mu\lambda}$ are called superpotentials and are functions of the metric tensor and its first order derivatives.

It is obvious that the energy-momentum complex is not uniquely determined by the condition that is usual divergence is zero since it can always been added to the energy-momentum complex a quantity with an identically vanishing divergence.

2 Einstein's Prescription

The energy-momentum complex of Einstein in a four-dimensional background is given as [1]

$$\theta_\nu^\mu = \frac{1}{16\pi} h_{\nu\ ,\lambda}^{\mu\lambda} \tag{10}$$

where the Einstein's superpotential $h_\nu^{\mu\lambda}$ is of the form

$$h_\nu^{\mu\lambda} = \frac{1}{\sqrt{-g}} g_{\nu\sigma} \left[-g\ \left(g^{\mu\sigma} g^{\lambda\kappa}\ -\ g^{\lambda\sigma} g^{\mu\kappa}\right)\right]_{,\kappa} \tag{11}$$

with the antisymmetric property

$$h_\nu^{\mu\lambda} = -h_\nu^{\lambda\mu}\ . \tag{12}$$

Thus, the energy and momentum in Einstein's prescription for a four-dimensional background are given by

$$P_\mu = \int\int\int \theta_\mu^0 dx^1 dx^2 dx^3 \tag{13}$$

and specifically the energy of the physical system in a four-dimensional background is

$$E = \int\int\int \theta_0^0 dx^1 dx^2 dx^3\ . \tag{14}$$

It should be noted that the calculations have to be restricted to the use of quasi-Cartesian coordinates.

In the case of two-dimensional gravitational backgrounds we have to modify expressions (10) and (14). Therefore, the energy-momentum complex for a physical system in Einstein's prescription for a two-dimensional gravitational background is given as

$$\theta_\nu^\mu = \frac{1}{4} h_{\nu\ ,\lambda}^{\mu\lambda} \tag{15}$$

and the energy is of the form

$$E = \int \theta_0^0 dx^1 \tag{16}$$

while the expression (11) for the superpotential remains the same.

If we are interested to evaluate the energy of a physical system in a two-dimensional gravitational background which is static, then the superpotentials $h_0^{0\lambda}$ are easily calculated

$$h_0^{00} = \frac{1}{\sqrt{-g}} g_{00} \left[-g \left(g^{00}g^{01} - g^{00}g^{01}\right)\right]_{,1} \tag{17}$$

$$h_0^{01} = \frac{1}{\sqrt{-g}} g_{00} \left[-g \left(g^{00}g^{11} - g^{10}g^{01}\right)\right]_{,1} \tag{18}$$

and after imposing staticity we get

$$h_0^{00} = 0 \tag{19}$$

$$h_0^{01} = \frac{1}{\sqrt{-g}} g_{00} \left[-g\, g^{00}g^{11}\right]_{,1} = 0 \ . \tag{20}$$

Therefore, for the case of two-dimensional static gravitational backgrounds the energy in Einstein's prescription will always be

$$\begin{aligned} E &= \frac{1}{4}\int h^{0\lambda}_{0\,,\lambda} dx^1 \\ &= \frac{1}{4}\int \left(h^{00}_{0\,,0} + h^{001}_{0\,,1}\right) dx^1 \\ &= 0 \ . \end{aligned} \tag{21}$$

It is obvious that the Einstein's prescription cannot be used in order to extract the energy distribution associated with two-dimensional static gravitational backgrounds.

3 Møller's Prescription

The energy-momentum complex of Møller in a four-dimensional background is given as [8]

$$\mathcal{J}_\nu^\mu = \frac{1}{8\pi}\xi_{\nu\ ,\lambda}^{\mu\lambda} \tag{22}$$

where the Møller's superpotential $\xi_\nu^{\mu\lambda}$ is of the form

$$\xi_\nu^{\mu\lambda} = \sqrt{-g}\left(\frac{\partial g_{\nu\sigma}}{\partial x^\kappa} - \frac{\partial g_{\nu\kappa}}{\partial x^\sigma}\right) g^{\mu\kappa} g^{\lambda\sigma} \tag{23}$$

with the antisymmetric property

$$\xi_\nu^{\mu\lambda} = -\xi_\nu^{\lambda\mu} \ . \tag{24}$$

Thus, the energy and momentum in Møller's prescription for a four-dimensional background are given by

$$P_\mu = \int\int\int \mathcal{J}_\mu^0 dx^1 dx^2 dx^3 \tag{25}$$

and specifically the energy of the physical system in a four-dimensional background is

$$E = \int\int\int \mathcal{J}_0^0 dx^1 dx^2 dx^3 \ . \tag{26}$$

Since we will be interested in two-dimensional gravitational backgrounds we have to modify expressions (22) and (26). Therefore, the energy-momentum complex for a physical system in Møller's prescription for a two-dimensional gravitational background is given as

$$\mathcal{J}_\nu^\mu = \frac{1}{2} \xi_{\nu\ ,\lambda}^{\mu\lambda} \tag{27}$$

and the energy is of the form

$$E = \int \mathcal{J}_0^0 dx^1 \tag{28}$$

while the expression (23) for Møller's superpotential remains the same.
It should be noted that the calculations are not anymore restricted to quasi-Cartesian coordinates but they can be utilized in any coordinate system.
If we are interested to evaluate the energy of a physical system in a two-dimensional gravitational background which is static, then the Møller's superpotentials $\xi_0^{0\lambda}$ are easily calculated

$$\xi_0^{00} = \sqrt{-g}\left(\frac{\partial g_{0\sigma}}{\partial x^\kappa} - \frac{\partial g_{0\kappa}}{\partial x^\sigma}\right) g^{0\kappa} g^{0\sigma} \tag{29}$$

$$\xi_0^{01} = \sqrt{-g}\left(\frac{\partial g_{0\sigma}}{\partial x^\kappa} - \frac{\partial g_{0\kappa}}{\partial x^\sigma}\right) g^{0\kappa} g^{1\sigma} \tag{30}$$

and after imposing staticity we get

$$\xi_0^{00} = 0 \tag{31}$$

$$\xi_0^{01} = -\sqrt{-g}\frac{\partial g_{00}}{\partial x^1}\, g^{00} g^{11} \ . \tag{32}$$

Therefore, for the case of two-dimensional static gravitational backgrounds the energy in Møller's prescription will always be

$$\begin{aligned} E &= \frac{1}{2}\int \xi_{0,\lambda}^{0\lambda} dx^1 \\ &= \frac{1}{2}\int \left(\xi_{0,0}^{00} + \xi_{0,1}^{01}\right) dx^1 \\ &= \frac{1}{2}\int \left[-\left(\sqrt{-g}\frac{\partial g_{00}}{\partial x^1}\, g^{00} g^{11}\right)_{,1}\right] dx^1 \ . \end{aligned} \tag{33}$$

It is obvious that the Møller's prescription can provide us with meaningful results for the energy distribution associated with two-dimensional static gravitational backgrounds.

4 "Schwarzschild" Black Hole

We start with the action in two spacetime dimensions [51]

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g}\, e^{-2\phi} \left[R + 4 \left(\nabla \phi \right)^2 + \lambda^2 \right] \tag{34}$$

where g is the determinant of the metric $g_{\mu\nu}(x)$ in two spacetime dimensions, ϕ is the dilaton field and λ^2 is the cosmological constant. This action arises as the effective action describing the radial modes of extremal dilatonic black holes in four or higher dimensions [52, 53, 54, 55, 56]. The black hole solution of (34) was given by E. Witten [46] as the low-energy approximation to an exact solution of string theory. The line element of the above-mentioned stringy two-dimensional "Schwarzschild" black hole solution can be written in different gauges (coordinate systems) as follows [57]:

i) "Schwarzschild" gauge

The two-dimensional dilatonic black hole in the "Schwarzschild" gauge is characterized by the line element:

$$ds^2 = -g(r)dt^2 + g^{-1}(r)dr^2 \tag{35}$$

where the function $g(r)$ is given by:

$$g(r) = 1 - \frac{M}{\lambda} e^{-2\lambda r} \tag{36}$$

and $0 < t < +\infty$, $r_H < r < +\infty$, with $r_H = \frac{1}{2\lambda} ln(\frac{M}{\lambda})$ the position of the event horizon of the black hole.

Following the terminology of the previous section, the covariant components of the metric function are

$$g_{00} = -\left(1 - \frac{M}{\lambda} e^{-2\lambda r} \right) \tag{37}$$

$$g_{11} = \frac{1}{\left(1 - \frac{M}{\lambda} e^{-2\lambda r}\right)} , \tag{38}$$

the corresponding contravariant components are

$$g^{00} = -\frac{1}{\left(1 - \frac{M}{\lambda} e^{-2\lambda r}\right)} \tag{39}$$

$$g^{11} = \left(1 - \frac{M}{\lambda} e^{-2\lambda r} \right) \tag{40}$$

and the determinant of the metric function $g_{\mu\nu}(x)$ is

$$g = -1 \ . \tag{41}$$

In order to evaluate the energy distribution in Møller's prescription associated with the exterior of the two-dimensional "Schwarzschild" black hole we evaluate the nonzero superpotential (32)

$$\xi_0^{01} = -2Me^{-2\lambda r} \tag{42}$$

and substituting it in equation (33) we get

$$\begin{aligned} E_{ext} &= \frac{1}{2}\int_R^{+\infty}\left[-\left(2Me^{-2\lambda r}\right)_{,r}\right]dr \\ &= -Me^{-2\lambda r}\Big|_R^{+\infty} \\ &= Me^{-2\lambda R} \ . \end{aligned} \tag{43}$$

Since it has been proved that the asymptotic value of the total gravitational mass of a two-dimensional "Schwarzschild" black hole is the mass parameter M which appear in the metric function (36) it is clear that the energy associated with a two-dimensional "Schwarzschild" black hole in a sphere of radius R will be

$$E(R) = M\left(1 - e^{-2\lambda R}\right) \ . \tag{44}$$

There are some comments in order. Firstly, the energy of two-dimensional "Schwarzschild" black hole is distributed to its interior as well as its exterior. Secondly, since the cosmological constant λ is positive, the energy distribution is positive everywhere, even in the forbidden region, i.e. $0 < r < r_H$. Therefore, a neutral test particle in the aforementioned gravitational background experiences at a finite radial distance a positive effective gravitational mass, given by equation (44). Thirdly, Seifert conjectured [50] that any singularity that occurs is hidden if a finite nonzero amount of matter tends to collapse into one point, or is naked if either one has singularities along lines (or surfaces) or the central singularities are carefully arranged that they contain zero mass. It is clear from equation (44) that at the origin, i.e. $R=0$, the mass of the two-dimensional "Schwarzschild" black hole is equal to zero providing a support to Seifert's hypothesis.

ii) Unitary gauge

The line element is:

$$ds^2 = -tanh^2(\lambda y)dt^2 + dy^2 \tag{45}$$

where the "unitary" variable y is given by the following expression:

$$y = \frac{1}{\lambda}ln\left[e^{\lambda(r-r_H)} + \sqrt{e^{2\lambda(r-r_H)} - 1}\right] \tag{46}$$

and $0 < y < +\infty$.

Following the terminology of Section 3, the covariant components of the metric function are

$$g_{00} = -tanh^2(\lambda y) \tag{47}$$
$$g_{11} = 1 \ , \tag{48}$$

the corresponding contravariant components are

$$g^{00} = -\frac{1}{tanh^2(\lambda y)} \tag{49}$$
$$g^{11} = 1 \tag{50}$$

and the determinant of the metric function $g_{\mu\nu}(x)$ is now given by

$$g = -tanh^2(\lambda y) \ . \tag{51}$$

In order to evaluate the energy distribution in Møller's prescription associated with the exterior of the two-dimensional "Schwarzschild" black hole but in the unitary gauge now, we evaluate the nonzero superpotential (32)

$$\xi_0^{01} = -\frac{2\lambda}{cosh^2(\lambda y)} \tag{52}$$

and substituting it in equation (33) we get

$$\begin{aligned} E_{ext} &= \frac{1}{2}\int_R^{+\infty}\left[-\left(\frac{2\lambda}{cosh^2(\lambda y)}\right)_{,y}\right]dy \\ &= \lambda\left[tanh^2(\lambda y) - 1\right]\Big|_R^{+\infty} \ . \end{aligned} \tag{53}$$

It is easily seen that since equations (37) and (47) are equal, the energy distribution in the unitary gauge (53) associated with the exterior of a two-dimensional "Schwarzschild" black hole is equal to the corresponding energy distribution derived in the Schwarzschild gauge (43) and therefore the energy associated with a two-dimensional "Schwarzschild" black hole in a sphere of radius R in the unitary gauge will be

$$E(R) = M\left(1 - e^{-2\lambda R}\right) \tag{54}$$

which is the same, as expected, with the corresponding expression (44) derived in the "Schwarzschild" gauge.

Obviously, the comments made in the "Schwarzschild" gauge also hold here.

iii) Conformal gauge

The line element in this gauge is:

$$ds^2 = (1 + e^{-2\lambda x})^{-1}(-dt^2 + dx^2) \tag{55}$$

where the variable x is given by:

$$x = \frac{1}{2\lambda} ln \left[e^{2\lambda(r - r_H)} - 1 \right] \tag{56}$$

and $-\infty < x < +\infty$.

Following as before the terminology of the Section 3, the covariant components of the metric function are

$$g_{00} = -\frac{1}{(1 + e^{-2\lambda x})} \tag{57}$$

$$g_{11} = \frac{1}{(1 + e^{-2\lambda x})} , \tag{58}$$

the corresponding contravariant components are

$$g^{00} = -\left(1 + e^{-2\lambda x}\right) \tag{59}$$

$$g^{11} = \left(1 + e^{-2\lambda x}\right) \tag{60}$$

and the determinant of the metric function $g_{\mu\nu}(x)$ is

$$g = -\frac{1}{(1 + e^{-2\lambda x})^2} . \tag{61}$$

In order to evaluate the energy distribution in Møller's prescription associated with the exterior of the two-dimensional "Schwarzschild" black hole we evaluate the nonzero superpotential (32)

$$\xi_0^{01} = -\frac{2\lambda e^{-2\lambda x}}{(1 + e^{-2\lambda x})} \tag{62}$$

and substituting it in equation (33) we get

$$\begin{aligned} E_{ext} &= \frac{1}{2}\int_R^{+\infty} \left[-\left(\frac{2\lambda e^{-2\lambda x}}{(1 + e^{-2\lambda x})} \right)_{,x} \right] dx \\ &= -\frac{\lambda e^{-2\lambda x}}{(1 + e^{-2\lambda x})} . \end{aligned} \tag{63}$$

After some, but not lengthy, calculations it is easily seen, using equations (56), (57) and the equality of the latter with (37), that the energy distribution in the conformal gauge

(63) associated with the exterior of a two-dimensional "Schwarzschild" black hole is equal to the corresponding energy distribution derived in the Schwarzschild gauge (43) and therefore the energy associated with a two-dimensional "Schwarzschild" black hole in a sphere of radius R in the conformal gauge will be

$$E(R) = M\left(1 - e^{-2\lambda R}\right) \tag{64}$$

which is the same, as expected, with the corresponding expressions (44) and (54) derived in the "Schwarzschild" and unitary gauge, respectively.
Obviously, the comments made in the "Schwarzschild" gauge also hold here.

5 "Reissner-Nordström" Black Hole

Our starting point in this section will be a two-dimensional effective action realized in heterotic string theory [48]

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 + 4\lambda^2 - \frac{1}{2}F^2\right] \tag{65}$$

where g is the determinant of the two-dimensional metric $g_{\mu\nu}(x)$, ϕ is the dilaton, λ^2 is the cosmological constant and $F_{\mu\nu}$ is the Maxwell stress tensor.
The line element of the charged two-dimensional black hole solution derived from (65) is given by (in coordinates corresponding to the "Schwarzschild" gauge of the previous section) [49]:

$$ds^2 = -g(r)dt^2 + g^{-1}(r)dr^2 \tag{66}$$

where

$$g(r) = 1 - \frac{M}{\lambda}e^{-2\lambda r} + \frac{Q^2}{4\lambda^2}e^{-4\lambda r} \tag{67}$$

with $0 < t < +\infty$, $r_+ < r < +\infty$, r_+ being the future event horizon of the black hole. The parameters M and Q are the mass and electric charge , respectively, of the "Reissner-Nordström" black hole (66-67) which is the corresponding charged black hole for the two-dimensional "Schwarzschild" black hole described in the previous section.
Following the terminology of the Section 3, the covariant components of the metric function are

$$g_{00} = -\left(1 - \frac{M}{\lambda}e^{-2\lambda r} + \frac{Q^2}{4\lambda^2}e^{-4\lambda r}\right) \tag{68}$$

$$g_{11} = \left(1 - \frac{M}{\lambda}e^{-2\lambda r} + \frac{Q^2}{4\lambda^2}e^{-4\lambda r}\right)^{-1} , \tag{69}$$

the corresponding contravariant components are

$$g^{00} = -\left(1 - \frac{M}{\lambda}e^{-2\lambda r} + \frac{Q^2}{4\lambda^2}e^{-4\lambda r}\right)^{-1} \tag{70}$$

$$g^{11} = \left(1 - \frac{M}{\lambda}e^{-2\lambda r} + \frac{Q^2}{4\lambda^2}e^{-4\lambda r}\right) \tag{71}$$

and the determinant of the metric function $g_{\mu\nu}(x)$ is

$$g = -1 \ . \tag{72}$$

In order to evaluate the energy distribution in Møller's prescription associated with the exterior of the two-dimensional "Reissner-Nordström" black hole we calculate the nonzero superpotential (32)

$$\xi_0^{01} = -2Me^{-2\lambda r} + \frac{Q^2}{\lambda}e^{-4\lambda r} \tag{73}$$

and substituting it in equation (33) we get

$$\begin{aligned} E_{ext} &= \frac{1}{2}\int_R^{+\infty}\left[-\left(2Me^{-2\lambda r} - \frac{Q^2}{\lambda}e^{-4\lambda r}\right)_{,r}\right] \\ &= \left[-Me^{-2\lambda r} + \frac{Q^2}{2\lambda}e^{-4\lambda r}\right]\Bigg|_R^{+\infty} \\ &= Me^{-2\lambda R} - \frac{Q^2}{2\lambda}e^{-4\lambda R} \ . \end{aligned} \tag{74}$$

It has been shown that the asymptotic value of the total gravitational mass of a two-dimensional "Reissner-Nordström" black hole is mass parameter M which appear in the metric function (67). Therefore, it is obvious that the energy associated with a two-dimensional "Reissner-Nordström" black hole in a sphere of radius R will be

$$E(R) = M - Me^{-2\lambda R} + \frac{Q^2}{2\lambda}e^{-4\lambda R} \ . \tag{75}$$

It is understood that the energy of the two-dimensional "Reisnner-Nordström" black hole is distributed to its interior as well as its exterior. Switching off the electric charge, i.e. $Q = 0$, we get that equation (75) goes over to (44). Additionally, treating λ as a positive constant, the energy distribution is positive everywhere, even in the forbidden region, i.e. $0 < r < r_H$ and therefore a neutral test particle in the aforementioned gravitational background experiences at a finite radial distance a positive effective gravitational mass, given by equation (75). Finally, it is easily seen from equation (75) that at the origin, i.e. $R{=}0$, the energy of the two-dimensional "Reissner-Nordström" black hole is nonzero and this is a counterexample[4] to Seifert's hypothesis [50].

[4]The case $Q = 0$ has been disregarded here since this is the "Schwarzschild" case.

6 Conclusions

In this work, we explicitly calculate the energy distributions associated with the two-dimensional "Schwarzschild" and "Reissner-Nordström" black holes. We, firstly, obtain the exact expressions of Møller's and Einstein's energy-momentum complexes in two-dimensional gravitational backgrounds. Although Møller's and Einstein's approaches associated with four dimensional gravitational backgrounds give meaningful results, in the case of two-dimensional static gravitational backgrounds Einstein's complex is proved to be always zero[5]. Therefore, since the two-dimensional "Schwarzschild" and "Reissner-Nordström" black holes are static backgrounds, we are led to work only with Møller's energy-momentum complex. We have found the explicit expression for the energy distribution associated with a two-dimensional "Schwarzschild" black hole. The result was extracted in three different coordinate systems (gauges) and found to be the same in all three gauges, verifying Møller's assertion that his expression of energy-momentum complex can be applied in any coordinate system giving meaningful outcomes. It is shown that the energy of the aforesaid stringy background is distributed to its interior as well as its exterior. Since this energy is positive everywhere, even behind the event horizon, i.e. r_H, a neutral test particle in this gravitational field "feels" a positive effective gravitational mass which is the energy that we have derived in Møller's approach. Concerning the two-dimensional "Reissner-Nordström" black hole analogous results are obtained. We calculate its energy distribution working only in the "Schwarzschild" gauge. Switching off the electric charge, i.e. $Q = 0$, we get the same result as in the case of the two-dimensional "Schwarzschild" black hole. Finally, it is noteworthy to observe that the energy distributions associated with the two-dimensional "Schwarzschild" and "Reissner-Nordström" black hole backgrounds provide an example and a counterexample, respectively, to the Seifert's hypothesis. We therefore agree with Virbhadra about the necessity of an adequate prescription for localization or quasi-localization of mass before discussing useful hypotheses within the framework of General Theory of Relativity.

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[5]The fact that Einstein's prescription fails to yield meaningful physical results in two-dimensional static gravitational backgrounds should not lead one to doubt about the validity of this approach in general.

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